

# Biased Contribution Index: A Simpler Mechanism to Maintain Fairness in Peer to Peer Network

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**Abstract**—To maintain fairness, in the terms of resources shared by an individual peer, a proper incentive policy is required in a peer to peer network. This letter proposes, a simpler mechanism to rank the peers based on their resource contributions to the network. This mechanism will suppress the free riders from downloading the resources from the network. Contributions of the peers are biased in such a way that it can balance the download and upload amount of resources at each peer. This mechanism can be implemented in a distributed system and it converges much faster than the other existing approaches.

**Index Terms**—Non-negative matrix, Eigenvector, Free Rider.

## I. INTRODUCTION

THE peers are motivated to share the resources in a peer to peer network, if they get at least the amount of data, what they have uploaded to the network. Most ideal situation is that, when upload and download amount for each peer is same. We call this, a fair situation in a peer to peer network. Of course, there will be no free riders in this situation. Therefore some mechanism is required to assure this. In this letter, we are proposing a simple mechanism called biased contribution index(BCI).

To achieve the aforementioned state, many incentive mechanisms have been studied [3], [4], [5], [6], [7]. Among these, global approaches [3], [4] performed better compared to local approaches [5], [6], [7]. In global approaches, shared history of peers in entire network is taken into consideration. It gives the wider view of peer's cooperation in the network. However, global approaches are not trivial to implement in the network. Like in [4], each peer needs to keep the record of every transaction history, regardless of whether he was directly involved in them or not. In [3], peer's contribution largely depends upon the contribution of peer with whom it is transacting so each peer needs to make some rough estimate of the impact on its contribution before each transaction. Our approach is similar to [3], but equal importance is given to the actual contribution of peer and the contribution of peer with whom it is transacting. It is simpler to implement in the network due to its faster convergence. With the suitable numerical example we compared the convergence result of our approach with [3]. We found that our approach performs far better in all the cases.

Rest of this paper is organized as follows. Section II presents the network model and introduction of biased contribution

index. Solution of biased contribution index is given in section III. Analysis of proposed algorithm is discussed in section IV. Section V shows the numerical results, and in section VI, conclusion is presented.

## II. NETWORK MODEL AND BIASED CONTRIBUTION INDEX

Let there be  $N$  peers in a peer to peer network. Peers share their resources with each other and their contribution is calculated globally. A simple metric which can best reflect the contribution of the peers in the network, could be the ratio of its total upload to the network to the total download from the network. But to motivate the peers to upload more to the peers contributing more and to download more from the peers contributing less, we need to bias this ratio by some incentive factor. Let this incentive factor be,  $x_i$ , for peer  $i$ . For peer  $i$ , we define, the upload to download ratio(biased ratio)as

$$R_i = \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}}.$$

Here,  $\mathbf{s}$  is  $N \times N$  share matrix. Its  $i j^{th}$  element represents the amount of resource shared by peer  $i$  to peer  $j$ .  $\mathbf{e}_i$  is the row vector with its  $i^{th}$  entry as '1' and all other entries as zero. The  $\mathbf{x}$  is a vector containing incentive factors. Let us define the incentive factor as a monotonically increasing function of biased ratio, i.e.

$$x_i = \frac{R_i}{1 + R_i},$$

$$= \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}}.$$

We call this incentive factor as biased contribution index (BCI).

Now to start the process of sharing we need to give some initial value of BCI to all the peers. Let us define a parameter  $\alpha \in (0, 1)$  to decide the initial value of BCI. Later we will see that the parameter  $\alpha$  is also related to the speed of convergence. The biased contribution index is modified to include this parameter  $\alpha$ , and is given by.

$$x_i = \begin{cases} \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}} + (1 - \alpha), & \text{if } \mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} \neq 0. \\ (1 - \alpha/2), & \text{otherwise.} \end{cases} \quad (1)$$

Here,  $(1 - \alpha/2)$  is the initial value of biased contribution index, when neither upload nor download has happened at the node. In the network of  $N$  nodes, there will be  $N$  unknowns and  $N$  nonlinear equations. We will see in the next section that, these equations can be solved by a suitable iterative function.

Peers are allowed to take the resources from network only if their biased contribution index is above a certain threshold

value. Therefore every peer will try to increase its biased contribution index, so that it can get the required amount of resources whenever needed.

It can be observed easily from equation 1 that a peer's biased contribution index will be higher if

- 1). Its contribution  $s_{ij}$  is higher,
  - 2). It shares more of its resources with higher contributing peer, and
  - 3). It takes more of the services from lower contributing peer.
- Therefore, intuitively we can say that this metric can assure fairness in the whole network. Later in section IV, we will give a mathematical justification for this.

### III. SOLUTION OF BIASED CONTRIBUTION INDEX

The BCI of any peer is expressed in the terms of the BCI of other peers. If  $\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} \neq 0$ , for  $i = 1, 2, \dots, N$ , then for  $N$  peers network, the equation 1, can be expressed in the form of matrix.

$$\mathbf{x} = \text{diag}[d_1, d_2, \dots, d_N] \cdot \mathbf{s} \cdot \mathbf{x} + (1 - \alpha) \mathbf{e}$$

Here  $d_i = \alpha / (\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x})$  and  $\mathbf{e}$  is vector with each element as '1'. We propose following Lemmas in this regard.

**Lemma 1.** The biased contribution index vector  $\mathbf{x} \in [(1 - \alpha), 1]^N$

*Proof.* When any peer  $i$  only takes the resources from the network and does not contribute any thing, then  $\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} = 0$  and  $\mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} \neq 0$ . In this case, biased contribution index of peer  $i$  will be minimum and it will be,

$$x_i = \alpha \cdot 0 + (1 - \alpha) = (1 - \alpha).$$

When a peer  $i$  only contribute the resources to the network without taking any thing, then  $\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} \neq 0$  and  $\mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} = 0$ . In this case, biased contribution index of peer  $i$  will be maximum and it will be,

$$x_i = \alpha \cdot 1 + (1 - \alpha) = 1.$$

In all other cases it will be in between these values. Hence  $\mathbf{x} \in [(1 - \alpha), 1]^N$ .  $\square$

**Lemma 2.** Let  $\mathbf{s}$  be  $N \times N$  non negative, irreducible matrix, then  $\mathbf{x}$  in the above expression can be calculated by the iterative function

$$\mathbf{x}^k = \phi(\mathbf{x}^{k-1}),$$

where  $i^{th}$  element of iterative function  $\phi(\mathbf{x}^{k-1})$  is  $\alpha[\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}^{k-1} / (\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}^{k-1} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}^{k-1})] + (1 - \alpha)$ .

*Proof.* The  $i^{th}$  element of iterative function  $\phi(\mathbf{x}^{k-1})$  is

$$x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}^{k-1}}{(\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}^{k-1} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}^{k-1})} + (1 - \alpha).$$

Let  $x_i^k$  and  $x_i^{k-1}$  are, far from actual solution  $x_i$  by  $\delta x_i^k$  and  $\delta x_i^{k-1}$  respectively, then

$$x_i + \delta x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})}{[\mathbf{e}_i \cdot \mathbf{s} \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1}) + \mathbf{e}_i \cdot \mathbf{s}^T \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})]} + (1 - \alpha).$$

Let  $\mathbf{s} + \mathbf{s}^T = \mathbf{s}'$ , then

$$x_i + \delta x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})}{\mathbf{e}_i \cdot \mathbf{s}' \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})} + (1 - \alpha).$$

$$\delta x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})}{\mathbf{e}_i \cdot \mathbf{s}' \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})} + (1 - \alpha) - x_i$$

Using equation 1,

$$\delta x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})}{\mathbf{e}_i \cdot \mathbf{s}' \cdot (\mathbf{x} + \delta \mathbf{x}^{k-1})} - \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}}.$$

$$\delta x_i^k = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}} \left[ \frac{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}})}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}})} - 1 \right].$$

It can be observed from equation 1,

$$x_i > \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}} = \alpha \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}};$$

hence,

$$\begin{aligned} \delta x_i^k &< x_i \left[ \frac{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}})}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}})} - 1 \right] \\ &= \frac{x_i}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}})} \left[ \left(1 + \frac{\mathbf{e}_i \cdot \mathbf{s} \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}\right) - \left(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}}\right) \right] \\ &= \frac{x_i}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}})} \left[ \left(\frac{\mathbf{e}_i \cdot \mathbf{s}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}\right) - \left(\frac{\mathbf{e}_i \cdot \mathbf{s}'}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}}\right) \right] \delta \mathbf{x}^{k-1} \\ &= \frac{1}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{s}' \cdot \delta \mathbf{x}^{k-1}}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}})} \left[ \left(\frac{x_i \mathbf{e}_i \cdot \mathbf{s}}{\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x}}\right) - \left(\frac{x_i \mathbf{e}_i \cdot \mathbf{s}'}{\mathbf{e}_i \cdot \mathbf{s}' \cdot \mathbf{x}}\right) \right] \delta \mathbf{x}^{k-1} \\ &= f_i(\delta \mathbf{x}^{k-1}) [\mathbf{A}_i - \mathbf{B}_i] \delta \mathbf{x}^{k-1} \end{aligned}$$

Where  $\mathbf{A}_i$  and  $\mathbf{B}_i$  are  $i^{th}$  row of  $N \times N$  matrix  $\mathbf{A}$  and  $\mathbf{B}$  respectively. It can be observed about matrix  $\mathbf{A}$  and  $\mathbf{B}$  that,  $\mathbf{A} \mathbf{x} = \mathbf{x}$  and  $\mathbf{B} \mathbf{x} = \mathbf{x}$ .

Matrix  $\mathbf{A}$  and  $\mathbf{B}$  are derived from matrix  $\mathbf{s}$ . Since matrix  $\mathbf{s}$  is irreducible hence matrix  $\mathbf{A}$  and  $\mathbf{B}$  will also be irreducible. Elements of vector  $\mathbf{x}$  are positive (see Lemma 1), so for non negative matrix  $\mathbf{s}$ , matrix  $\mathbf{A}$  and  $\mathbf{B}$  will also be non negative. Therefore spectral radius of matrix  $\mathbf{A}$  and  $\mathbf{B}$  will be '1' and corresponding eigen vector will be  $\mathbf{x}$  (see [8]).

If  $\delta \mathbf{x}^{k-1} \ll \mathbf{x}$ , then  $f_i(\delta \mathbf{x}^{k-1}) \approx 1$ . Hence,

$$\delta \mathbf{x}^k < [\mathbf{A} - \mathbf{B}] \delta \mathbf{x}^{k-1} < [\mathbf{A} - \mathbf{B}]^k \delta \mathbf{x}^0$$

$$\lim_{k \rightarrow \infty} \delta \mathbf{x}^k < \lim_{k \rightarrow \infty} [\mathbf{A} - \mathbf{B}]^k \delta \mathbf{x}^0 = 0$$

(see Theorem 1 in [1])

And if  $\delta \mathbf{x}^{k-1} > \mathbf{x}$ , then  $f_i(\delta \mathbf{x}^{k-1}) < 1$ . Hence in this case,  $\delta x_i^{k-1}$  will decrease more rapidly till  $\delta \mathbf{x}^{k-1} \ll \mathbf{x}$ .

Hence  $\mathbf{x}$  can be calculated by the aforementioned iterative function.  $\square$

**Lemma 3.** If  $\mathbf{x} = a\mathbf{e}$  is the solution of biased contribution index then  $a$  will be  $(1 - \alpha/2)$ .

*Proof.* If  $\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} \neq 0$ , equation 1 can be written as

$$\begin{aligned} x_i(\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}) &= \alpha \mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + (1 - \alpha)(\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}) \\ \Rightarrow x_i(\mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + \mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x}) &= \mathbf{e}_i \cdot \mathbf{s} \cdot \mathbf{x} + (1 - \alpha)\mathbf{e}_i \cdot \mathbf{s}^T \cdot \mathbf{x} \end{aligned}$$

for  $i = 1, 2, \dots, N$ . Hence, above relation can be written in the form of matrix as follows.

$$\text{diag}(\mathbf{x})(\mathbf{s} + \mathbf{s}^T)\mathbf{x} = \mathbf{s}\mathbf{x} + (1 - \alpha)\mathbf{s}^T\mathbf{x}.$$

Here  $\text{diag}(\mathbf{x})$  is  $N \times N$  diagonal matrix with its  $ii^{th}$  element as  $x_i$ . Now if  $\mathbf{x} = a\mathbf{e}$  then

$$\begin{aligned} a\mathbf{I}(\mathbf{s} + \mathbf{s}^T)a\mathbf{e} &= a\mathbf{s}.\mathbf{e} + (1 - \alpha)a\mathbf{s}^T.\mathbf{e} \\ \Rightarrow a^2(\mathbf{s} + \mathbf{s}^T)\mathbf{e} &= a(\mathbf{s}.\mathbf{e} + (1 - \alpha)\mathbf{s}^T.\mathbf{e}) \end{aligned}$$

Pre-multiply by  $\mathbf{e}^T$  on both side

$$\begin{aligned} a^2\mathbf{e}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{e} &= a(\mathbf{e}^T.\mathbf{s}.\mathbf{e} + (1 - \alpha)\mathbf{e}^T.\mathbf{s}^T.\mathbf{e}) \\ \Rightarrow a^2(\mathbf{e}^T.\mathbf{s}.\mathbf{e} + \mathbf{e}^T.\mathbf{s}^T.\mathbf{e}) &= a(\mathbf{e}^T.\mathbf{s}.\mathbf{e} + (1 - \alpha)\mathbf{e}^T.\mathbf{s}^T.\mathbf{e}) \end{aligned}$$

for any matrix  $\mathbf{s}$ ,  $\mathbf{e}^T.\mathbf{s}.\mathbf{e}$  will be the sum of all of its elements. Hence  $\mathbf{e}^T.\mathbf{s}.\mathbf{e} = \mathbf{e}^T.\mathbf{s}^T.\mathbf{e} = T$ , and above expression can be written as

$$a^2(T + T) = a(T + (1 - \alpha)T)$$

since  $a \in [(1 - \alpha), 1]$ , hence

$$a(2T) = (2 - \alpha)T$$

since  $T \neq 0$ , hence  $a = (1 - \alpha/2)$   $\square$

**Lemma 4.** If  $\mathbf{x} = (1 - \alpha/2)\mathbf{e}$  is the solution of biased contribution index then

$$(\mathbf{s}^T - \mathbf{s})\mathbf{e}_i^T \perp \mathbf{e} \quad \forall i.$$

*Proof.* Substituting  $\mathbf{x} = (1 - \alpha/2)\mathbf{e}$  in equation 1

$$\begin{aligned} (1 - \alpha/2) &= \alpha \frac{(1 - \alpha/2)\mathbf{e}_i.\mathbf{s}.\mathbf{e}}{(1 - \alpha/2)(\mathbf{e}_i.\mathbf{s}.\mathbf{e} + \mathbf{e}_i.\mathbf{s}^T.\mathbf{e})} + (1 - \alpha) \\ \Rightarrow \alpha/2 &= \alpha \frac{\mathbf{e}_i.\mathbf{s}.\mathbf{e}}{(\mathbf{e}_i.\mathbf{s}.\mathbf{e} + \mathbf{e}_i.\mathbf{s}^T.\mathbf{e})} \\ \Rightarrow \alpha(\mathbf{e}_i.\mathbf{s}.\mathbf{e} + \mathbf{e}_i.\mathbf{s}^T.\mathbf{e}) &= 2\alpha\mathbf{e}_i.\mathbf{s}.\mathbf{e} \\ \Rightarrow \alpha\mathbf{e}_i(\mathbf{s} - \mathbf{s}^T).\mathbf{e} &= 0 \\ \Rightarrow \alpha((\mathbf{s}^T - \mathbf{s})\mathbf{e}_i^T)^T.\mathbf{e} &= 0 \end{aligned}$$

Since  $\alpha \neq 0$ , hence

$$(\mathbf{s}^T - \mathbf{s})\mathbf{e}_i^T \perp \mathbf{e} \quad \forall i$$

**Lemma 5.** If  $\mathbf{se} = \mathbf{s}^T\mathbf{e}$ , then the biased contribution index vector,  $\mathbf{x} = (1 - \alpha/2)\mathbf{e}$ .

*Proof.*  $\mathbf{se} = \mathbf{s}^T\mathbf{e} \Rightarrow \mathbf{e}^T\mathbf{s}^T = \mathbf{e}^T\mathbf{s}$ , now from equation 1, if  $\mathbf{e}_i.\mathbf{s}.\mathbf{x} + \mathbf{e}_i.\mathbf{s}^T.\mathbf{x} \neq 0$ .

$$\text{diag}(\mathbf{x})(\mathbf{s} + \mathbf{s}^T)\mathbf{x} = \mathbf{s}\mathbf{x} + (1 - \alpha)\mathbf{s}^T\mathbf{x}$$

Pre-multiply by  $\mathbf{e}^T$  on both side

$$\begin{aligned} \mathbf{e}^T\text{diag}(\mathbf{x})(\mathbf{s} + \mathbf{s}^T)\mathbf{x} &= \mathbf{e}^T\mathbf{s}\mathbf{x} + (1 - \alpha)\mathbf{e}^T\mathbf{s}^T\mathbf{x} \\ \Rightarrow \mathbf{x}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{x} &= \mathbf{e}^T\mathbf{s}\mathbf{x} - (\alpha/2)\mathbf{e}^T\mathbf{s}^T\mathbf{x} + (1 - \alpha/2)\mathbf{e}^T\mathbf{s}^T\mathbf{x} \\ \Rightarrow \mathbf{x}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{x} &= \mathbf{e}^T\mathbf{s}\mathbf{x} - (\alpha/2)\mathbf{e}^T\mathbf{s}\mathbf{x} + (1 - \alpha/2)\mathbf{e}^T\mathbf{s}^T\mathbf{x} \\ \Rightarrow \mathbf{x}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{x} &= (1 - \alpha/2)\mathbf{e}^T\mathbf{s}\mathbf{x} + (1 - \alpha/2)\mathbf{e}^T\mathbf{s}^T\mathbf{x} \end{aligned}$$

$$\Rightarrow \mathbf{x}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{x} = (1 - \alpha/2)\mathbf{e}^T(\mathbf{s} + \mathbf{s}^T)\mathbf{x}$$

$$\Rightarrow [\mathbf{x}^T - (1 - \alpha/2)\mathbf{e}^T](\mathbf{s} + \mathbf{s}^T)\mathbf{x} = 0$$

It is clear that  $(\mathbf{s} + \mathbf{s}^T)$  is non negative matrix and  $\mathbf{x} > 0$  hence  $(\mathbf{s} + \mathbf{s}^T)\mathbf{x} \neq 0$ . Hence  $\mathbf{x} = (1 - \alpha/2)\mathbf{e}$ .  $\square$

#### IV. ANALYSIS OF BIASED CONTRIBUTION INDEX

##### A. Solution Of Free Riding and Collusion

Initially every peer is allowed to take some resources from the network; otherwise process of sharing will not start. Hence, initial BCI of  $(1 - \alpha/2)$  for each peer is justified. But as soon as BCI is updated, free rider's BCI will reach at minimum level. Because for any free rider  $i$ ,  $\mathbf{e}_i.\mathbf{s}.\mathbf{x} = 0$ , hence from equation 1,  $x_i = (1 - \alpha)$ . This will disqualify them from taking any resources from the network in future, until they acquire sufficient BCI.

The contributing peer will always gain the BCI and resource taking peer will always loose the BCI. Hence, peers will always avoid reporting the false transaction and thus, collusion can be avoided in the network.

##### B. Justification For Fairness

We can observe the following from the above discussion.

- 1) If the biased contribution index of all peers are same, then the amount of resources contributed will be same as what is taken from the network for each peer.
- 2) If resources contributed and resources taken from the network in each peer are same, then the biased contribution index of all peers will be same.

First point is evident from section III. That is, if biased contribution index of all peers are same then it will be  $(1 - \alpha/2)$  (see Lemma 3). And if biased contribution index will be  $(1 - \alpha/2)$  then  $(\mathbf{s}^T - \mathbf{s})\mathbf{e}_i^T \perp \mathbf{e}$  for all  $i$  (from Lemma 4). Hence

$$\begin{aligned} \mathbf{e}_i.(\mathbf{s} - \mathbf{s}^T).\mathbf{e} &= 0 \quad \forall i \\ \Rightarrow \mathbf{e}_i.\mathbf{s}.\mathbf{e} &= \mathbf{e}_i.\mathbf{s}^T.\mathbf{e} \quad \forall i \end{aligned}$$

Hence total amount of resources contributed and taken from the network by each peer will be same.

Second point can be understood directly from Lemma 5. If resources contributed and taken from the network in each peer are same then

$$\mathbf{e}_i.\mathbf{s}.\mathbf{e} = \mathbf{e}_i.\mathbf{s}^T.\mathbf{e} \quad \forall i$$

Hence  $\mathbf{se} = \mathbf{s}^T\mathbf{e}$  and  $\mathbf{x} = (1 - \alpha/2)\mathbf{e}$ . Hence biased contribution index of all peers will be same.

##### C. Implementation In Distributed System

Calculation of biased contribution index can be implemented in a distributed system following the same approach as in [1] [2] [3]. Multiple other peers named index managers, can be assigned to maintain the record of biased contribution index of any peer. Whenever any peer needs the biased contribution index, of other peers it can send the query to the respective index managers. If there is any conflict about the biased contribution index of a peer, it can be settled, by majority of voting

Table I: Biased Global Contribution index for,  $\alpha = 0.8$  in each iteration

$i$	1	2	3	4
$x^0$	0.6000	0.6000	0.6000	0.6000
$x^1$	0.7440	0.5000	0.5333	0.6174
$x^2$	0.7266	0.4823	0.5161	0.6373
$x^3$	0.7202	0.4861	0.5170	0.6379
$x^4$	0.7207	0.4870	0.5177	0.6371
$x^5$	0.7210	0.4869	0.5177	0.6370
$x^6$	0.7210	0.4868	0.5177	0.6370
$x^7$	0.7210	0.4868	0.5177	0.6370

Table II: Biased Global Contribution index for,  $\alpha = 0.4$  in each iteration

$i$	1	2	3	4
$x^0$	0.8000	0.8000	0.8000	0.8000
$x^1$	0.8720	0.7500	0.7667	0.8087
$x^2$	0.8690	0.7465	0.7634	0.8124
$x^3$	0.8685	0.7468	0.7634	0.8124
$x^4$	0.8686	0.7468	0.7635	0.8124
$x^5$	0.8686	0.7468	0.7635	0.8124

by index managers. In this way we can avoid the collusion among peers. For calculation of biased contribution index of any peer, index manager needs to know the contribution and resource taken by that peer and biased contribution index of peer with whom it is transacting. Calculation is repeated till the convergence of biased contribution index is achieved. If number of iterations required to converge the algorithm are less, then required number of update messages will also be less. Therefore the algorithm can be implemented with less overhead. In next section, we will compare the speed of convergence of our method with the other algorithm [3].

## V. NUMERICAL RESULT

We considered the share matrix  $s$  as

$$\begin{bmatrix} 0 & 100 & 50 & 20 \\ 20 & 0 & 30 & 40 \\ 10 & 40 & 0 & 50 \\ 50 & 10 & 60 & 0 \end{bmatrix}.$$

### A. Speed of Convergence

The number of iterations required for convergence of the biased contribution index were estimated for two different values of  $\alpha$ . For  $\alpha = 0.8$ , BCI in each step is shown in Table I. We can see that it converges in seven iterations. For  $\alpha = 0.4$  (see Table II), it converge only in five iterations. Thus impact of  $\alpha$  is clearly evident in the results.

### B. Comparison of Convergence Speed With GC[3]

We compared the number of iterations required for convergence of the biased contribution index and global contribution [3]. In latter case, we have taken the different values of  $\alpha$  and  $\beta$  as defined in [3]. Results are shown in Table III. We can observe that the number of iterations required for convergence of BCI is always lesser than the global contribution mentioned in [3].

Table III: Number of iterations, required to converge the BCI and GC [3] for different values of  $\alpha$  and  $\beta$ (parameter  $\beta$  is defined as in [3])

	Number of iteration required in GC[3]	Number of iteration required in BCI
$\alpha = 0.9$	$\beta = 0.8, \text{Iterations} = 9$	$\text{Iterations} = 8$
	$\beta = 0.5, \text{Iterations} = 10$	
	$\beta = 0.2, \text{Iterations} = 10$	
$\alpha = 0.8$	$\beta = 0.8, \text{Iterations} = 10$	$\text{Iterations} = 7$
	$\beta = 0.5, \text{Iterations} = 8$	
	$\beta = 0.2, \text{Iterations} = 11$	
$\alpha = 0.7$	$\beta = 0.8, \text{Iterations} = 9$	$\text{Iterations} = 7$
	$\beta = 0.5, \text{Iterations} = 8$	
	$\beta = 0.2, \text{Iterations} = 9$	
$\alpha = 0.6$	$\beta = 0.8, \text{Iterations} = 8$	$\text{Iterations} = 6$
	$\beta = 0.5, \text{Iterations} = 7$	
	$\beta = 0.2, \text{Iterations} = 8$	
$\alpha = 0.5$	$\beta = 0.8, \text{Iterations} = 7$	$\text{Iterations} = 5$
	$\beta = 0.5, \text{Iterations} = 6$	
	$\beta = 0.2, \text{Iterations} = 8$	
$\alpha = 0.4$	$\beta = 0.8, \text{Iterations} = 7$	$\text{Iterations} = 5$
	$\beta = 0.5, \text{Iterations} = 6$	
	$\beta = 0.2, \text{Iterations} = 7$	
$\alpha = 0.3$	$\beta = 0.8, \text{Iterations} = 6$	$\text{Iterations} = 4$
	$\beta = 0.5, \text{Iterations} = 5$	
	$\beta = 0.2, \text{Iterations} = 6$	
$\alpha = 0.2$	$\beta = 0.8, \text{Iterations} = 5$	$\text{Iterations} = 3$
	$\beta = 0.5, \text{Iterations} = 5$	
	$\beta = 0.2, \text{Iterations} = 6$	

## VI. CONCLUSION

In this work, we propose a new metric, the biased contribution index, to evaluate the contributions of the peers in the network. Using this metric we can discourage the free riding in the network. We can also ensure the balance between the total upload and download by a node in the network. We compared our method with another existing approach [3]. With the help of numerical example, we have shown that our metric converges in lesser number of iterations compared to the global contribution approach given in [3]. Our approach can also be implemented in a distributed system and is much simpler than the other existing approach.

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